The Research Basis for the Assessing Math Concepts (AMC) Series of Assessments for Grades K–3 Mathematics

Our Commitment

At Math Perspectives, our programs are founded on the latest scientific research on how students learn most effectively. We work closely with students and classrooms to ensure that our programs are pedagogically sound and reflect current and scientifically proven instructional methodologies. We are committed to providing materials and strategies for teachers that represent the best and most current research-based findings.

What is formative assessment?

Formative assessments are systematic and regular measurements of students' progress in the classroom and the process by which the results are used to inform instructional decisions and practice. Assessment results are used by teachers to guide their instruction to meet students' learning needs. The primary objective of a formative assessment is to guide instruction rather than measure where students are at the end of an instructional period. When used regularly, formative assessments provide essential information for the planning and delivery of instruction.

Formative Assessment Research Findings

In 1998, Paul Black and Dylan Wiliam of Kings College, London, reviewed the international literature on assessment for evidence that the use of formative (classroom) assessments raises student achievement. They uncovered more than 250 relevant research articles. The information they pooled on the estimated effects of formative assessment on summative test scores revealed an unprecedented positive effect on student achievement—effect sizes of between a half and full standard deviation. Such effect sizes equate to percentile score gains of 15 to 30 points, or three or more years in grade equivalents (Stiggins, 2002).

Black and Wiliam (1998a) found firm evidence that formative assessment is an essential component of classroom work and that its use can raise standards of achievement. More than 20 of the 250 studies they reviewed showed that the practice of regular formative assessment, in conjunction with instruction based on individual needs, produced significant learning gains.

These studies incorporated a range of age groups from five-year-olds through university undergraduates. The average improvement in the test scores of the students who experienced formative assessment was compared with the range of scores found among groups of students who had not experienced formative assessment. Typical effect sizes for the formative assessment experimental groups were between 0.4 and 0.7. These effect sizes are larger than most of those found for educational interventions (Black and Wiliam, 1998b).

Assessing Math Concepts Research Application: Continuous Formative Assessment

Kathy Richardson developed the *Assessing Math Concepts* (AMC) series of formative assessments to assist teachers in providing continuous formative assessment in their classrooms.

Research has shown that learning about number and operations is a complex process for children (e.g., Fuson, 1992). Kathy Richardson developed the *Assessing Math Concepts* (AMC) series of formative assessments to assist teachers in providing continuous formative assessment in their classrooms.

This series of nine formative assessments for prekindergarten through third-grade mathematics focuses on the core concepts that must be in place for children to understand and be successful in mathematics. The series is based on the premise that teachers will be able to provide more effective instruction when they are aware of the Critical Learning Phases for mathematics and of students' progress within these phases.

The AMC assessments:
- Are aligned to the Critical Learning Phases.
- Pinpoint what children know and still need to know.
- Document children's growth over time.

The nine assessments and books are:

Book /Assessment	**Goal**
Counting Objects	To determine whether a child can: • count and keep track of an unorganized pile of up to 32, and • make a pile of up to 18 counters.
Changing Numbers	To determine whether a child: • knows if a number is larger or smaller than another number, and • can change one quantity into another.

Book /Assessment	Goal
More/Less Trains	To determine whether a child can: • use one train of counters to figure out another, and • compare trains and /or piles to find out how many more or less one quantity is than the other.
Number Arrangements	To determine whether a child can recognize parts of a number and combine those parts without having to count all.
Combination Trains	To determine what number combinations a child knows and whether the child can use related combinations as a strategy for getting answers.
Hiding Assessment	To determine what number combinations a child knows by determining whether they can tell the missing part of a number without having to figure it out.
Ten Frames	To determine whether a child can combine numbers by making a 10 and leftovers, and subtract using known combinations and relationships.
Grouping Tens	To determine whether a child: • can tell "how many" in a quantity if the number of tens and ones is known • can add 10 and take away 10 without counting • knows that the total number does not change when counted in a different way • understands what it means to count by twos and fives, and to find out how well the child can do this.
Two-Digit Addition & Subtraction	To determine: • whether a child can subtract from two-digit numbers by mentally breaking numbers apart and reorganizing them into tens and leftovers, and • how the child solves problems presented symbolically.

Research Finding: Formative assessment is most effective when followed by instruction.

Thomas R. Guskey (1997, 2003) states that "assessments must be followed by high-quality, corrective instruction designed to remedy whatever learning errors the assessment identified." Guskey further states, "To charge ahead knowing that students have not learned certain concepts or skills well would be foolish. Teachers must therefore follow their assessments with instructional alternatives that present those concepts in new ways and engage students in different and more appropriate learning experiences. High-quality, corrective instruction is not the same as reteaching" (2003).

AMC Research Application: The Critical Learning Phases

A phase is a particular moment or stage in a process, especially one at which a significant development occurs or a particular condition is reached. Kathy Richardson has defined the Critical Learning Phases for grades K–3 mathematics students as:

"Assessments must be followed by high quality, corrective instruction designed to remedy whatever learning errors the assessment identified."
— Thomas R. Guskey (1997, 2003)

- The essential ideas that are milestones or hurdles in children's growth of understanding.
- The developments that determine the way a child is able to think with numbers and use numbers to solve problems.
- The understandings that must be in place to ensure that children are not just imitating procedures or saying words they do not really understand (illusions of learning).

The AMC assessments follow the stages of children's growth in the Critical Learning Phases of the core concepts for numbers to 100. The assessments pinpoint what children know and still need to learn and can be used to document children's growth over time.

The extent of young children's knowledge and understanding can never be fully determined through paper and pencil tasks. Teachers get much more complete and useful information when they watch and interact with their students while they are doing mathematical tasks. How the students respond indicates what Critical Learning Phase they have reached and reveals their level of understanding. Indicators that describe the range of responses and identify students' instructional needs are listed on each of the assessment forms and are explained in detail in each book in the *Assessing Math Concepts* series.

Because number concepts develop in relatively predictable ways, the majority of children at specific grade levels need to work with particular concepts and

Critical Learning Phases. Teachers can use the AMC assessments to determine the instructional needs of their students along a continuum as they develop an understanding of these particular concepts. Each book in the series includes a set of grade-level charts that guide teachers in using the assessments to plan appropriate whole-class instruction as they focus on various concepts throughout the year.

In addition, each book contains a section entitled "Linking Assessment and Instruction" that teachers can use after they have identified the needs of their students.

Classroom teachers can use the AMC assessments to determine the level of thinking a child has reached. Knowing this level will guide their instruction and ensure that the child is working at the level that is most appropriate, therefore ensuring maximum growth and understanding.

Research Finding: High quality formative assessment is effective; however, the effectiveness is dependent on the quality of the test items.

Black and Wiliam (1998) provide extensive and detailed research to support the use of formative assessment to ensure student learning. Their summary statements provide guidance for educators regarding the use of formative assessment and include the following: *Frequent short tests are better than infrequent long ones, and the quality of the test items is essential.*

AMC Research Application: One-on-One Student Interviews

The AMC assessments are carried out in short one-on-one interviews that can be conducted with students on an ongoing basis to guide instruction. Using student results, explicit instructional activities are suggested for each level. Each assessment captures detailed information about the student's understanding of core concepts. These assessments were developments through extensive field tests with the following school districts and projects:

Clark County School District, Las Vegas, NV

- MASE (Mathematics and Science Enhancement) – Project teachers
- MAPS (Math Assessment for Primary Schools) – Selected schools involved in the field-testing of the assessment forms
- Sue Plummer and Lori Squires, Project leaders

Visalia Unified School District, Visalia, CA

- STEPS – Math coaches, teacher leaders, and classroom teachers

Larabee Elementary School, Bellingham, WA

- Cathy Young, Math coach

Childs Elementary School, Bloomington, IN

- Chris Oster, Teacher

Edmonds School District, Edmonds, WA

- Edmonds Math Project – Teacher leaders

Math Perspectives' Work with Three School Districts

This section summarizes three school districts' partnerships with Math Perspectives and their respective results using the AMC assessments in grades K–3 classrooms. It can be concluded from this work that when teachers receive ongoing professional development and use the AMC formative assessments, they are able to establish the individual and collective instructional needs of their students and can adjust their instruction accordingly.

San Diego City Schools Primary Mathematics Initiative: Building Professional Development Around Children's Thinking

Profile: San Diego City Schools, San Diego, California

- 132,482 students (PreK–12)
- 2nd largest district in California
- 8th largest urban district in the United States
- 9,044 certificated staff:
  - Kindergarten teachers……....605
  - First-grade teachers……....643
  - Second-grade teachers……....654
- 114 elementary schools
- Ethnic diversity (students):
  - Hispanic……..43.3%
  - White ……25.6%
  - Asian……...16.6%
  - African American……...13.9%
- 27.5% English learners
- 54% eligible for free or reduced meals

Professional Development Focus and Design

The San Diego City Schools Math Department teamed with Math Perspectives to provide ongoing professional development to the district's teachers of K–2 mathematics. This initiative asked the following questions:

- What are the foundational mathematical ideas young children need to know in order to have access to standards?
- How do we know that children are learning these important ideas?
- How do we teach children so that they learn these important ideas and meet the standards?

The professional development was delivered according to the following design:

- The program director and district math coaches attended Math Perspectives' week-long assessment institute and follow-up sessions throughout the year.
- All primary teachers attended four (4) trainings throughout the year.
- Teachers learned about the foundational ideas being assessed.
- Teachers learned to administer the AMC assessments.
- Teachers went back to their classrooms and each assessed six (6) students.
- Teachers returned to the next session with assessment results.

The following AMC assessments were used in the grade level specified:

- Kindergarten: *Counting Objects*, *Changing Numbers*, and *Number Arrangements*
- First Grade: *Combination Trains*, *Hiding Assessment*, and *Ten Frames*
- Second Grade: *Ten Frames*, *Grouping Tens*, and *Two-Digit Addition & Subtraction*

"Teachers began teaching children rather than teaching curriculum. ... Prior to this, teachers taught the curriculum page by page without thinking about children's understanding."
—Brian Tash, San Diego City Schools

Outcomes

Brian Tash, District Program Manager for Elementary Mathematics during the project, noted the following outcomes of the initiative:

1. Primary teachers began having conversations about how young children learn and truly became communities of learners.
2. Teachers began teaching children rather than teaching curriculum. Teachers began planning according to what their students knew and understood. Prior to this, teachers taught the curriculum page by page without thinking about children's understanding.
3. Benchmark assessments showed improvement in mathematical fluency (accuracy, efficiency, flexibility).
4. Though the focus on mathematics decreased with a new superintendent, teachers who were engaged in improving their practice continued to work with the assessments and guide their instruction based on student needs.

The chart below summarizes the test scores for San Diego City Schools second-grade students before and after the Primary Mathematics Initiative. Before the initiative, 33% of second-grade students tested "below basic or far below basic" in mathematics. After district leaders and classroom teachers participated in the Math Perspectives professional development and implemented the AMC assessment series to guide their instruction, the percentage of second-graders who tested "below basic or far below basic" decreased to 19%.

Systemic Treatment				
	No systemic treatment	**P.D.—Curriculum Materials**	**No systemic treatment**	**P.D.—Assessing Math Concepts**
Year tested	2001–2002	2002–2003	2003–2004	2004–2005
Students tested	11,614	11,186	10,693	10,003
% Enrolled	96%	98%	97.7%	97.9%
% Advanced or proficient	41%	50%	52%	61%
% Basic	25%	24%	25%	20%
% Below basic or far below basic	33%	26%	23%	19%

Aurora Publics Schools, Aurora, Colorado: Working in Partnership With Math Perspectives Since 1999

Profile: Aurora Publics Schools

- 31,643 students as of October 1, 2007 (PreK–12)
- More than 2,000 certificated staff
- 32 elementary schools
- Ethnic diversity (students):
  Hispanic.....51.6%
  White.....22.6%
  Black.....20.8%
  Asian.....4.1%
  Native American.....0.9%
- 37% of students are English learners. These students speak 84 different languages, with 89% of this group being Spanish speakers.
- 59% eligible for free or reduced meals

Professional Development Design

Math Perspectives and the Aurora Public Schools have worked together since 1999 to provide professional development to the district's instructional specialists, district math coaches, administrators, building math coaches, and classroom teachers of K–6 mathematics.

Many district administrators, math coaches, and classroom teachers have attended Math Perspectives Leadership Institutes and follow-up sessions from 1999 to the present. Each year the District provides Math Perspectives professional development for teachers and administrators, including the week-long Assessing Math Concepts Institute.

Teachers of K–6 mathematics in the district use the AMC series of assessments along with the *Developing Number Concepts* and *Understanding Numbers* curriculum materials by Kathy Richardson.

Colorado Student Assessment Program (CSAP)

The CSAP tests are designed to measure student achievement in relationship to the Model Content Standards established by the Colorado Department of Education. These standards specify what students should know at designated grade levels. The following chart summarizes the Aurora Public School students' increases in math achievement on the CSAP test administered in the 2005–06 school year, compared with CSAP math scores for the 2004–05 school year.

CSAP Results for the Aurora Public Schools 2005–06 School Year

Elementary School	Middle School	High School
10% increase in 3rd grade math	3% increase in 6th grade math	4% increase in 9th grade math
6% increase in 4th grade math	3% increase in 8th grade math	

Source: Colorado Student Assessment Program

Quality Teaching in Mathematics Project: Sable and Montview Elementary Schools, Aurora Public Schools, Aurora, Colorado

The Aurora Public Schools Quality Teaching in Mathematics project was a three-year project (2002–2005) for the professional development of teachers and

district coaches. The third and final school year of the project was funded by the Jay and Rose Phillips Family Foundation.

The Quality Teaching in Mathematics project focused on the continued development of Montview Elementary and Sable Elementary schools as math education demonstration schools and on training district-level math coaches for capacity building throughout the district. The project was designed to support the development of the schools as learning organizations for teachers by making professional development part of the workday, providing innovative teacher training in mathematics, raising standards for the teaching of mathematics, and measuring both teacher and student performance.

Professional Development Design

Teachers, in collaboration with Math Perspectives consultants, evaluated formative data in order to differentiate instruction and drive decision-making.

- Throughout the school year, teachers involved in the project at each site met with both building and district coaches to evaluate the formative data and determine areas of strength and improvement for students. A variety of instructional strategies were discussed and teachers developed differentiated instructional plans based on their students' individual needs. Student progress was monitored.
- Montview teachers in grades 3–5 participated in math academies, facilitated by in-building teacher leaders, in which formative assessment data was evaluated to create unit and daily plans.
- Montview and Sable teachers in grades K–2 worked with the Math Perspectives consultants to deepen their understandings of the Kathy Richardson Number Sense assessments. As a group, they evaluated these assessments and used the results to help them plan for independent work.
- Sable teachers worked with the Math Perspectives consultants to implement independent and small group workstations in grades K–5.
- District instructional coordinators, district math coaches, and building teacher leaders participated in two assessment institutes facilitated by Math Perspectives consultants. The group developed an understanding of how to use the results of these assessments to guide instruction in their own classrooms as well as to help develop the understanding of the teachers they support.

The Role of the Math Perspectives Consultants

During the 2004–05 school year, Math Perspectives participated as follows:

1. Math Perspectives consultants provided in-service training for principals at two assessment institutes.
2. Montview and Sable teachers attended summer workshops on Assessing Number Concepts, Teaching for Understanding, and Thinking with Numbers. After working with the Montview and Sable teachers, the Math Perspectives consultants continued coursework for teachers as a supplement to their monthly visits.
3. Additional demonstration classrooms were developed at both Montview and Sable Elementary Schools.

Outcomes

The following chart represents the Montview and Sable teachers' self-assessment of their understanding of the AMC assessments following their training.

Understanding the AMC Assessments	Montview Post-test	Sable Post-test
	Agree or Strongly Agree (N = 12)	Agree or Strongly Agree (N = 20)
I understand the Critical Learning Phases.	92%	55%
I understand how the assessments are linked to the Critical Learning Phases.	83%	65%
I regularly administer the AMC assessments.	66%	45%
I use the AMC assessments to plan for instruction (whole group, small group).	92%	75%
I use the AMC assessments to plan for independent work.	75%	55%
I understand how to use the AMC assessments to differentiate instruction for my students.	83%	80%
I understand how to use the AMC assessments to plan for Number Talks.	75%	85%

Case Studies of Student Growth

Building math teachers selected students to monitor through yearlong case studies. The teachers in both schools focused on students who needed to accelerate in order to meet or exceed grade-level expectations. Most of the students were at least one year behind in academic achievement. Teachers' case studies indicated that the students' understanding of numbers increased because the teachers had a better understanding of how children learn mathematics. The teachers' responses suggested that growth in their own content knowledge had an impact on their students' success.

"I learned that although I believe in giving children time and opportunity to practice, I actually do not give them enough time to internalize concepts about numbers."
—Teacher, Aurora, CO, Public Schools

The following quotes are from the teacher-leader case studies:

"I never knew how complex counting was. Through watching (my student) struggle as he counted past 10, I realized it's not just landing on one number after another, rather, its understanding."

"Watching (my student) develop an understanding of and a need for tens and ones has been fascinating."

"I need to be aware of the Critical Learning Phases a child needs to go through and then make sure that I provide multiple opportunities for all my students to enter and exit those phases when they are ready and when they see a need."

"I now understand the phases students go through in order to think with numbers and use numbers to solve problems."

"I learned that although I believe in giving children time and opportunity to practice, I actually do not give them enough time to internalize concepts about numbers."

Teachers also noted that students' affect and attitudes toward mathematics and school improved as their mathematics skills developed:

"(My student's) attendance has improved greatly."

"Students work much harder on something that they feel successful at."

Case Studies of Teacher Growth

The principals of the Montview and Sable schools were asked to reflect on the growth of the teachers as a result of their participation in Mathematical Perspectives professional development. Both of the principals observed improvement in math instruction. Their reflections included the following:

- The focus this year has been on number sense development through the Critical Learning Phases, number talks, and independent work. The teachers have a deeper understanding of number sense and the appropriate resources for student work.

- Teachers see themselves as having increased their understanding of the Critical Learning Phases. This has allowed them to have more specific "look fors" as they monitor students at work and to more clearly observe what their students understand and what their next steps are. This ability to closely monitor students at work has allowed the teachers to ask more focused questions that accelerate student learning.

- Teachers indicated that their increased ability to assess their students' number sense understandings has allowed them to plan more intentionally and more efficiently. They are better able to differentiate their instruction by choosing specific groupings, approaches, and resources. This focused planning has ensured that all students have opportunities to work at their point of need.

Principal Growth

Through their work with the Math Perspectives consultants, both principals indicated that they have more knowledge of the AMC assessments and that they feel more confident that they can support their teachers in using these assessments.

Paradise Professional Development School, Clark County Schools, Las Vegas, Nevada: What Types of Relationships and Trends Exist in the Student Interview Mathematics Assessment?

In the fall of 2006 every K–5 student enrolled at Paradise Professional Development School was given grade-level specific mathematics pre-assessments, through individual interviews, that reflected conceptual understandings in number from counting to place value. These qualitative assessments provided rich and meaningful data that allowed teachers to make purposeful instructional decisions in their classrooms. The assessment responses not only assisted the classroom teachers but were converted into quantitative data that was used to report relationships, correlations, patterns, and trends in the students' conceptual understanding of number. Preliminary findings suggested that particular mathematics concepts have significant relationships and that specific number concepts are grasped at a particular age or grade level.

Profile: Paradise Professional Development School

Paradise Professional Development School is a preK–5 elementary school located on the University of Nevada Las Vegas campus in the East Region of the Clark County School District. Paradise is a Title One school and one of two professional development schools in the district. The school offers free breakfast and lunch to the students. The experience level of classroom teachers ranges from 1 to 24 years of teaching.

- 777 students (preK–5)
- Average student-to-teacher ratio: 21:1
- Ethnic diversity (students)
  - Hispanic……45.1%
  - White ……...27.9%
  - African American…….17.0%
  - Asian/Pacific Islander…….8.7%
  - American Indian/Alaskan Native……..1.3%
- 81% eligible for free or reduced meals

Sample Size and Study Methods

A sample of 494 K–5 elementary students in 28 classrooms was used in the study. The students were interviewed individually by trained University of Las Vegas mathematics tutors or by the mathematics specialist. The interviews were taken from the mathematics assessments in the AMC series as follows:

Kindergarten: *Counting Using Objects*, *Small Number Relationships*
Grade 1: *Counting Using Objects*, *Small Number Relationships*, *Number Combinations to 10*
Grades 2–5: *Number Combinations to 10*, *Place Value*

An additional assessment was administered in third grade that related to understanding the concept of multiplication. Concrete models such as Unifix Cubes and/or base ten blocks were used by the students to demonstrate their thinking in solving the mathematics problems posed by the interviewer.

Findings

One of the mathematics assessments administered to 197 kindergarten and first-grade students focused on adding 1 and subtracting 1 to a given number. For example, if a student can count up to 38, the interviewer will ask, "What if you have 35 cubes and I add one more. How many will there be?" or "What if you have 38 cubes and I take one away. How many are left?"

As summarized in the chart below (Figure 1), the data shows that on average the students had slightly more success with adding 1 to a given number than taking 1 away from a given number. The mean for adding 1 was 2.03 (needs practice) while the mean for subtracting 1 was 1.63 (needs instruction).

Statistics for Kindergarten and 1st Grade Students

		Add 1	Subtract 1
N	Valid	197	197
	Missing	297	297
Mean		2.03	1.63
Median		2.00	1.00
Mode		3	1

Figure 1

The data in the next chart (Figure 2) reveals a moderate positive relationship between the concepts of adding 1 to and subtracting 1 from a given number.

Correlations for Adding 1 and Subtracting 1

		Add 1	Subtract 1
Add 1	Pearson Correlation	1	.430**
	Sig. (2-tailed) N		.000 197
		197	
Sub 1	Pearson Correlation	.430**	1

Figure 2

** Correlations between .10 and .30 are referred to as low positive relationships, .40 and .60 as moderate positive relationships, and .70 and above as high positive relationships (McMillan, 2004, p. 134).

The following histograms (Figures 3–4) show the number and quality of responses that kindergarten and first-grade students made when adding 1 to a given number.

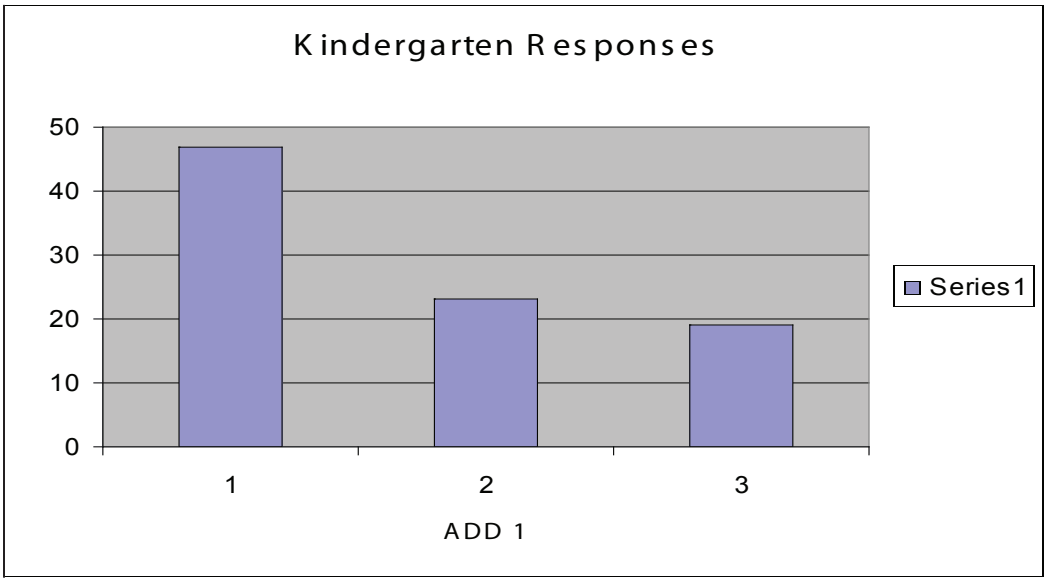

Figure 3

1 = Needs Instruction 2 = Needs Practice 3 = Ready to Apply

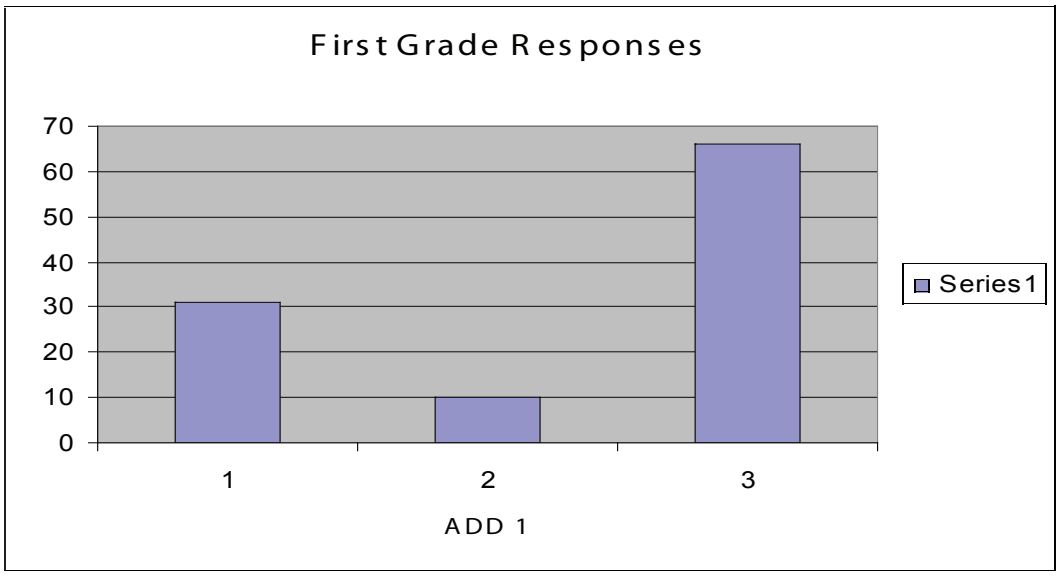

Figure 4

1 = Needs Instruction 2 = Needs Practice 3 = Ready to Apply

Of the 89 kindergarten students assessed, 47 received "needs instruction," 23 received "needs practice," and 19 received "ready to apply." Of the 109 first-grade students assessed, 31 received "needs instruction," 10 received "needs practice," and 66 received "ready to apply." The trend that seems to appear is that kindergarten students are at a stage at the beginning of the year where they need instruction in adding 1 whereas first-graders are able to apply the concept of adding 1 to a given number.

One of the mathematics assessments, the *Hiding Assessment*, was administered to first through fifth-graders. It shows the number a student can internalize by hiding a particular number of cubes from a given set. For example, if a student can internalize to 7, that would mean the student knows all the number combinations of 5 and 2, 6 and 1, 4 and 3, and so on without hesitation. The following charts (Figures 5–9) reveal in the mean the average number that students internalize in the Hiding Assessment.

First Grade Statistics

		Hiding Asssessment
N	Valid	107
	Missing	0
Mean		5.0841
Median		5.0000
Mode		5.00

Figure 5

The average number that first-graders internalize is to 5.

Second Grade Statistics

		Hiding Asssessment
N	Valid	77
	Missing	0
Mean		6.2597
Median		6.0000
Mode		5.00

Figure 6

The average number that second-graders internalize is to 6.

Third Grade Statistics

		Hiding Asssessment
N	Valid	83
	Missing	1
Mean		7.6627
Median		8.0000
Mode		10.00

Figure 7

The average number that third-graders internalize is to 7.

Fourth Grade Statistics

		Hiding Asssessment
N	Valid	60
	Missing	0
Mean		8.4000
Median		8.0000
Mode		10.00

Figure 8

The average number that fourth-graders internalize is to 8.

Fifth Grade Statistics

		Hiding Asssessment
N	Valid	77
	Missing	0
Mean		8.9481
Median		10.0000
Mode		10.00

Figure 9

The average number that fifth-graders internalize is to 8.

The pattern that appears in the charts above reveals that as the grade level increases, the internalization number also increases by 1 except in fifth grade, where the students internalized to 8.9 instead of 9. Often the internalization is lower than most educators predict, because they believe that students have fluency with numbers to 10 and are ready to work with larger numbers with understanding.

The mathematics assessment *Grouping Tens* was administered to 285 second through fifth-graders and focused on place value, adding and subtracting 10, and conservation of number. For example, the interviewer asks:

- "If you have 3 piles of 10 and 8 leftover, how many will there be altogether?" (Place value)
- "How many cubes will you have if we add 10 to 38?" (Adding 10)
- "How many cubes will you have if we take 10 away from 38?" (Subtracting 10)
- "If we push the small piles of cubes back together in one pile, then how many cubes will there be?" (Conservation of number).

Grouping Ten Assessment
(Correlations between Adding 10, Subtracting 10, Place Value, and Conservation of Number)

		Adding 10	Subtract-ing 10	Place Value	Conser-vation
Adding 10	Pearson Correlation	1	.952**	.704**	.488**
	Sig. (2-tailed) N	285	.000 285	.000 285	.000 279
Subtracting 10	Pearson Correlation	.952**	1	.693**	.448**
	Sig. (2-tailed) N	.000 285	285	.000 285	.000 279
Place Value	Pearson Correlation	.704**	.693**	1	.554**
	Sig. (2-tailed) N	.000 285	.000 285	287	.000 280
Conserva-tion	Pearson Correlation	.488**	.448**	.554**	1
	Sig. (2-tailed) N	.000 279	.000 279	.000 280	281

Figure 10

Correlations between .10 and .30 are referred to as low positive relationships, .40 and .60 as moderate positive relationships, and .70 and above as high positive relationships (McMillan, 2004, p. 134).

The data reveals that there is a high positive relationship between adding 10 to and subtracting 10 from a given number, yet there were only two codings for this concept (1 = Yes, 0 = No). In the adding and subtracting of 1 in the Counting Assessment, the correlation was moderate (see Figure 2) yet there were 3 codings (Needs Improvement = 1, Needs Practice = 2, Ready to Apply = 3).

The concept of place value relates to thinking of numbers as composed of tens and ones, and the correlation is slightly higher in adding 10, at .704, than subtracting 10, at .693. When students understand the concept of conservation of number, they know that the essential properties are constant even when the superficial properties change. For example, 3 piles of 10 is 30 even when it is grouped into 5 piles of 6. The conservation correlation is moderately positive in adding and subtracting 10, at .488, and in the place value task, at .554.

The four assessed concepts in *Grouping Tens* are slightly different in their make-up, but appear to have some inherent similarities in terms of conceptual understandings. Several of the questions in the assessments are scaffolded in order to get to the heart of the child's mathematical thinking.

Conclusions

Several overall conclusions can be drawn, with justification, from the types of relationships and trends that emerged from the mathematics interview assessment data for the 494 K–5 students at Paradise Professional Development School. It appears that:

- Kindergarten and first-grade students had slightly more success in adding 1 to a number than subtracting 1 from a number. Adding 1 is often the first relationship that is learned by primary students followed by the concept of one less. Students later begin to think about the actual quantity and how it relates to other quantities in a comparison between numbers (Richardson, 2002, pp. 43–44).

- There is a moderate positive relationship between the concepts of adding 1 and subtracting 1 to a number. The concept of adding 1 and subtracting 1 frequently refers to number relationships and developing a sense of quantity. Students become more fluent in developing these beginning number relationships through concrete experiences where they are asked to change the quantity. The two concepts are related as they are often

taught simultaneously in the classroom and are experienced in real-life situations.

- There is a trend, as kindergarten students are at a Needs Instruction stage in adding 1 whereas first-grade students are at a Ready to Apply stage in adding 1. Kindergarten students are developing the concept of counting and this is where most of their work in mathematics takes place. Developing number relationships such as adding 1 comes from a solid base of counting and knowing numbers, which is often found in first-grade classrooms. The majority of kindergarten classrooms ground the work in counting in order to move students towards understanding numbers and their relationships to one another.

- There is a pattern in the internalization number; as the grade increases, so does the internalization number. The internalization number is typically associated with the age of the child, and the movement from one number to another is small and complex. Understanding all the parts of numbers to 10 with fluency and ease is a difficult concept to grasp, and showing an increase in the internalization number from one grade level to another is expected with appropriate experiences in composition and decomposition of number.

- There is a moderate-to-high relationship between place value, adding and subtracting 10, and conservation of number. The concept of place value relies on the basic understanding that one ten is the same as ten ones, a concept that is related to conservation of number. Knowing the composition of tens and ones, students often can add 10 and subtract 10 with ease and fluency and not rely on counting.

The study described above was conducted by Michelle R. Adams, Academic Specialist – Math, Paradise Professional Development School, Clark County School District, Las Vegas, Nevada.

Reflections, Lessons Learned, and Suggestions for Future Research

The purpose of the research presented in this paper is to demonstrate the need to change our current practice of teaching mathematics to students. We believe that by showing what students know or lack using performance-based interviews and specific grade-level assessments, educators will look more deeply at the current practice of teaching and learning mathematics.

In conducting this work, we knew that qualitative research was meaningful and depicted the thinking of a child much more accurately than a number alone, but we also considered our current culture—quantitative data would need to be a part of the work. To keep a balance between the two forms of research, we converted the qualitative data into quantitative data but shared the anecdotal responses of the students with the classroom teacher. The quantitative data would be reported, yet we would use qualitative language to support the numbers.

Constant reflection on the work of this action research project has led us in improving our research for the future. The most important lesson learned from the work is that the vision must focus on students. The work is about implementing best practices that improve student achievement. It is important to note that the data collected and analyzed thus far centers on the mathematical thinking of students; it does not show the growth in mathematical understanding of the students studied. Because of time constraints, the post-data being collected will need to be analyzed for evidence of student growth and reported in future research.

Kathy Richardson

Kathy Richardson is one of the nation's leading elementary mathematics educators. She is a student of children's thinking who has worked to support the improvement of schools for 40 years. As a former classroom teacher, she has focused on bringing best practices in mathematics to the classroom through the development of classroom and professional development materials, including assessments. She is passionate about making schools good places for all students and believes in the need for solutions that go beyond creating "illusions of learning" and, rather, look carefully at what children need to know and understand to be successful.

Her *Developing Number Concepts* series is used by teachers and school districts nationwide. Today, she is the Program Director of the Math Perspectives Teacher Development Center in Bellingham, Washington, which she cofounded.

Conclusion

The *Assessing Math Concepts* formative assessments are supported by research and are designed to help teachers provide individual instruction for every student, based on each student's individual needs.

The AMC assessments are based on the premise that teachers will be able to provide more effective instruction and ensure maximum learning for each of their students when they are aware of the essential steps children move through as they develop an understanding of foundational mathematical ideas. The AMC assessments cover all of the important steps in this learning process and provide detailed information for instructional decision-making.

Kathy Richardson strongly states that teachers must follow up the information they receive from the assessments with the appropriate learning experiences for each student.

Math Perspectives is committed to demonstrating that its programs can help teachers and students in the classroom. We will continue to monitor progress in schools that provide ongoing professional development using our teaching resources and strategies.

New Reporting Technology: AMC Anywhere

Math Perspectives has teamed with Didax, Inc. in Rowley, Massachusetts to develop the AMC Anywhere software. AMC Anywhere makes the Assessing Math Concepts more efficient for teachers to put into practice. Using a Palm® Handheld device, teachers have the flexibility to assess students anywhere and can take advantage of immediate results and comprehensive reporting.

AMC Anywhere:

- Reduces time and improves the efficiency of administering assessments.
- Accurately reflects students' understanding.
- Automatically summarizes instructional needs to tailor instruction.
- Provides web-based reporting of student and classroom information to drive instruction and impact student achievement.

AMC Anywhere provides a rich and accessible body of data that can be quickly analyzed, distilled, and distributed through the dedicated AMC website.

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